Postbuckling Behavior of a Thick Plate

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The postbuckling behavior of a simply supported rectangular thick plate subjected to arbitrary loading conditions is investigated. The applied stress is taken to be a combination of a pure bending stress plus an extensional stress in the plane of the plate. Governing equations based on von Kármán assumptions are used to solve the postbuckling problem by the Galerkin method. Buckling loads are obtained to compare with the results of Brunelle and Robertson. Postbuckled deflection is shown to increase with the transverse isotropic coefficient S, and the effects of the in-plane bending stress are found to be quite significant for a thick plate.

Nomenclature

a,b	= plate lengths in x and y directions, respectively
D^*	= modulus of rigidity of plate
E	= modulus of elasticity
G	= shear modulus
G^*	= transverse shear modulus
h	= plate thickness
K	= in-plane compressive stress coefficient, $K=12b^2N_x/(\pi^2h^2D)$
M_x,M_y,M_{xy}	= bending moment
N_x,N_y,N_{xy},N_z	= stress resultant
r	= aspect ratio, $r=a/b$
S	= transverse isotropic parameter, $S=Dh/b^2G^*=Eh^2/G^*b^2(1-\nu)$
u_1,u_2	= in-plane displacement
w	= transverse deflection, and $W=w/h$
x_1,x_2,x_3	= Cartesian coordinates
β	= ratio of bending stress to normal stress, $\beta=\sigma_m/\sigma_n$
κ^2	= shear correction factor, $\kappa^2=\pi^2/12$
ν	= Poisson's ratio, $\nu=0.30$
ρ	= density
ψ_1,ψ_2	= angular changes of lines initially normal to the neutral surface

Introduction

It is well known that an elastic flat plate subjected to in-plane compressive loading undergoes a primary buckling from an initial undeflected equilibrium state. This primary buckling state is generally a stable symmetric branching point and, in the postprimary buckling range, this plate is able to carry a greater load as the amplitude of deflection increases. The postbuckling behavior of rectangular plates has been the subject of a great deal of study for many years. Numerous theoretical analyses[1-8] have been formulated and a variety of experimental investigations[9-11] have been undertaken to examine the phenomena. All of these investigators were concerned primarily with in-plane compressive stresses and most of their studies applied the thin-plate theory. Research work dealing with the postbuckling behavior of a transversely isotropic thick plate in an arbitrary state-of-stress field is not found in the literature.

Brunelle and Robertson[12] used the average stress method and the variational method to derive the linear equations of motion for a thick plate in an arbitrary state of initial stress.

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The authors have derived the nonlinear equation[13] on the basis of von Kármán's assumption by using the average stress method to study the large-amplitude vibration problem.

In the present work, previously derived equations are used to study the postbuckling behavior. These equations are solved by the Galerkin approximate method. The postbuckling problems of a simply supported thick rectangular plate subjected to in-plane normal and bending stresses are studied and the influences of various parameters of postbuckling behavior are investigated.

The Governing Equations

We shall consider a body in a state of nonuniform initial stress in static equilibrium and subjected to a time-varying incremental deformation. Following a technique described by Bolotin,[14] Brunelle and Robertson[12] derived the following equations by using the perturbing technique,

$$(\sigma_{ij}\bar{u}_{s,j})+[\bar{\sigma}_{ij}(\delta_{sj}+u_{s,j}+\bar{u}_{s,j})]_{,i}+\Delta X_s+\bar{X}_s-\rho\ddot{\bar{u}}_s=0 \quad (1)$$

$$\bar{P}_s+\Delta P_s=[\sigma_{ij}\bar{u}_{s,j}+\bar{\sigma}_{ij}(\delta_{js}+u_{s,j}+\bar{u}_{s,j})]n_i \quad (2)$$

The incremental displacements are assumed to be of the following

$$\bar{u}_1(x_1,x_2,x_3,t)=u_1(x_1,x_2,t)+x_3\psi_1(x_1,x_2,t) \quad (3)$$

$$\bar{u}_2(x_1,x_2,x_3,t)=u_2(x_1,x_2,t)+x_3\psi_2(x_1,x_2,t) \quad (4)$$

$$\bar{u}_3(x_1,x_2,t)=w(x_1,x_2,t) \quad (5)$$

It is assumed that the initial displacement gradients are so small that the product $\bar{\sigma}_{ij}u_{s,j}$ can be neglected. For the plate theory of large deflection the von Kármán assumptions are employed. Therefore, the perturbated displacement gradients are also so small that the terms $\bar{\sigma}_{ij}\bar{u}_{s,j}$ may be dropped except for $\bar{\sigma}_{i1}\bar{u}_{3,1}$, and $\bar{\sigma}_{i2}\bar{u}_{3,2}$. By using stress-displacement relations and the average stress method, the nonlinear governing equations of a transversely isotropic thick rectangular plate derived by the authors[13] will be employed to study the postbuckling phenomena.

In the equations of motion to follow, the nomenclature is

$$N_x=\int\sigma_{11}dx_3,\qquad N_y=\int\sigma_{22}dx_3$$

$$N_z=\int\sigma_{33}dx_3,\qquad N_{xy}=\int\sigma_{12}dx_3$$

$$Q_x=\int\sigma_{13}dx_3,\qquad Q_y=\int\sigma_{23}dx_3$$

$$Q_x^*=\int x_3\sigma_{13}dx_3,\qquad Q_y^*=\int x_3\sigma_{23}dx_3$$

$$M_x=\int\sigma_{11}x_3dx_3,\qquad M_y=\int\sigma_{22}x_3dx_3,\qquad M_{xy}=\int\sigma_{12}x_3dx_3$$

$$M_x^*=\int\sigma_{11}x_3^2dx_3,\qquad M_y^*=\int\sigma_{22}x_3^2dx_3,\qquad M_{xy}^*=\int\sigma_{12}x_3^2dx_3$$

$$D=Eh/(1-\nu^2) \quad D^*=Eh^3/12(1-\nu^2) \quad Gh^3/12=D^*(1-\nu)/2$$

$$\Delta F_{nn}=\int\Delta P_n \mathrm{d}x_3, \qquad \bar{F}_{nn}=\int\bar{P}_n \mathrm{d}x_3$$

$$\Delta M_{nn}=\int\Delta P_n x_3 \mathrm{d}x_3, \qquad \bar{M}_{nn}=\int\bar{P}_n x_3 \mathrm{d}x_3$$

$$\Delta F_{nt}=\int\Delta P_t \mathrm{d}x_3, \qquad \bar{F}_{nt}=\int\bar{P}_t \mathrm{d}x_3$$

$$\Delta M_{nt}=\int\Delta P_t x_3 \mathrm{d}x_3, \qquad \bar{M}_{nt}=\int\bar{P}_t x_3 \mathrm{d}x_3$$

$$\Delta F_{n3}=\int\Delta P_3 \mathrm{d}x_3, \qquad \bar{F}_{n3}=\int\bar{P}_3 \mathrm{d}x_3$$

where all integrals are from $-h/2$ to $h/2$.

The x_1 extension equation is

$$D(u_{1,1}+\tfrac{1}{2}w_{,1}^2+\nu u_{2,2}+\nu\tfrac{1}{2}w_{,2}^2)_{,1}+Gh(u_{1,2}+u_{2,1}+w_{,1}w_{,2})_{,2}$$
$$+(N_x u_{1,1}+M_x\psi_{1,1}+N_{xy}u_{1,2}+M_{xy}\psi_{1,2}+\psi_1 Q_x)_{,1}$$
$$+(N_y u_{1,2}+M_y\psi_{1,2}+N_{xy}u_{1,1}+M_{xy}\psi_{1,1}+\psi_1 Q_y)_{,2}+f_x=\rho h\ddot{u}_1 \tag{6}$$

where

$$f_x=\int_{-h/2}^{h/2}(\bar{X}_1+\Delta X_1)\mathrm{d}x_3+u_{1,1}(\sigma_{31}^+-\sigma_{31}^-)$$
$$+\frac{h}{2}\psi_{1,1}(\sigma_{31}^++\sigma_{31}^-)+u_{1,2}(\sigma_{32}^+-\sigma_{32}^-)+\frac{h}{2}\psi_{1,2}(\sigma_{32}^++\sigma_{32}^-)$$
$$+\psi_1(\sigma_{33}^+-\sigma_{33}^-)+\bar{\sigma}_{31}^+-\bar{\sigma}_{31}^-$$

The (+) and (−) imply that the stresses are evaluated at the top and bottom faces of the plate, respectively. The stresses with an overbar are due to the load causing the incremental deformation. Similar definitions hold for the remaining equations.

The x_2 extension equation is

$$Gh(u_{1,2}+u_{2,1}+w_{,1}w_{,2})_{,1}+D(u_{2,2}+\nu u_{1,1}+\tfrac{1}{2}w_{,2}^2+\tfrac{1}{2}\nu w_{,1}^2)_{,2}$$
$$+(N_x u_{2,1}+M_x\psi_{2,1}+N_{xy}u_{2,2}+M_{xy}\psi_{2,2}+Q_x\psi_2)_{,1}$$
$$+(N_y u_{2,2}+M_y\psi_{2,2}+N_{xy}u_{2,1}+M_{xy}\psi_{2,1}+Q_y\psi_2)_{,2}+f_y=\rho h\ddot{u}_2 \tag{7}$$

where

$$f_y=\int_{-h/2}^{h/2}(\bar{X}_2+\Delta X_2)\mathrm{d}x_3+u_{2,1}(\sigma_{31}^+-\sigma_{31}^-)$$
$$+\frac{h}{2}\psi_{2,1}(\sigma_{31}^++\sigma_{31}^-)+u_{2,2}(\sigma_{32}^+-\sigma_{32}^-)+\frac{h}{2}\psi_{2,2}(\sigma_{32}^++\sigma_{32}^-)$$
$$+\psi_2(\sigma_{33}^+-\sigma_{33}^-)+\bar{\sigma}_{32}^+-\bar{\sigma}_{32}^-$$

The shear force equation is

$$D(\epsilon_1+\nu\epsilon_2)w_{,11}+2Gh\gamma w_{,12}+D(\nu\epsilon_1+\epsilon_2)w_{,22}$$
$$+D(\epsilon_1+\nu\epsilon_2)_{,1}w_{,1}+Gh\gamma_{,2}w_{,1}+D(\nu\epsilon_1+\epsilon_2)_{,2}w_{,2}$$
$$+Gh\gamma_{,1}w_{,2}+\kappa^2G^*h[(\psi_1+w_{,1})_{,1}+(\psi_2+w_{,2})_{,2}]$$
$$+(N_x w_{,1}+N_{xy}w_{,2})_{,1}+(N_{xy}w_{,1}+N_y w_{,2})_{,2}+q=\rho h\ddot{w} \tag{8}$$

where

$$q=\int_{-h/2}^{h/2}(\bar{X}_3+\Delta X_3)\mathrm{d}x_3+w_{,1}(\sigma_{31}^+-\sigma_{31}^-)+w_{,2}(\sigma_{32}^+-\sigma_{32}^-)$$
$$+\bar{\sigma}_{33}^+-\bar{\sigma}_{33}^-+w_{,1}(\bar{\sigma}_{31}^+-\bar{\sigma}_{31}^-)+w_{,2}(\bar{\sigma}_{32}^+-\bar{\sigma}_{32}^-)$$

$$\epsilon_1=u_{1,1}+\tfrac{1}{2}w_{,1}^2, \quad \epsilon_2=u_{2,2}+\tfrac{1}{2}w_{,2}^2, \quad \gamma=u_{1,2}+u_{2,1}+w_{,1}w_{,2}$$

The x_1 moment equation is

$$D^*(\psi_{1,1}+\nu\psi_{2,2})_{,1}+\frac{D^*}{2}(1-\nu)(\psi_{1,2}+\psi_{2,1})_{,2}$$
$$+(M_x u_{1,1}+M_x^*\psi_{1,1}+M_{xy}u_{1,2}+M_{xy}^*\psi_{1,2}+Q_x^*\psi_1)_{,1}$$
$$+(M_{xy}u_{1,1}+M_{xy}^*\psi_{1,1}+M_y u_{1,2}+M_y^*\psi_{1,2}+Q_y^*\psi_1)_{,2}$$
$$+(M_{xy}u_{1,1}+M_{xy}^*\psi_{1,1}+M_y u_{1,2}+M_y^*\psi_{1,2}+Q_y^*\psi_1)_{,2}$$
$$-Q_x u_{1,1}-Q_x^*\psi_{1,1}-Q_y u_{1,2}-Q_y^*\psi_{1,2}-N_z\psi_1$$
$$-\kappa^2G^*h(\psi_1+w_{,1})+m_x=\frac{\rho h^3}{12}\ddot{\psi}_1 \tag{9}$$

where

$$m_x=\int_{-h/2}^{h/2}(\bar{X}_1+\Delta X_1)x_3\mathrm{d}x_3+\frac{h}{2}\Big[u_{1,1}(\sigma_{31}^++\sigma_{31}^-)$$
$$+\frac{h}{2}\psi_{1,1}(\sigma_{31}^+-\sigma_{31}^-)+u_{1,2}(\sigma_{32}^++\sigma_{32}^-)+\frac{h}{2}\psi_{1,2}(\sigma_{32}^+-\sigma_{32}^-)$$
$$+\psi_1(\sigma_{33}^++\sigma_{33}^-)+\bar{\sigma}_{13}^++\bar{\sigma}_{13}^-\Big]$$

The x_2 moment equation is

$$\frac{D^*}{2}(1-\nu)(\psi_{1,2}+\psi_{2,1})_{,1}+D^*(\psi_{2,2}+\nu\psi_{1,1})_{,2}$$
$$+(M_x u_{2,1}+M_x^*\psi_{2,1}+M_{xy}u_{2,2}+M_{xy}^*\psi_{2,2}+Q_x^*\psi_2)_{,1}$$
$$+(M_{xy}u_{2,1}+M_{xy}^*\psi_{2,1}+M_y u_{2,2}+M_y^*\psi_{2,2}+Q_y^*\psi_2)_{,2}$$
$$-Q_x u_{2,1}-Q_x^*\psi_{2,1}-Q_y u_{2,2}-Q_y^*\psi_{2,2}-N_z\psi_2$$
$$-\kappa^2G^*h(\psi_2+w_{,2})+m_y=\frac{\rho h^3}{12}\ddot{\psi}_2 \tag{10}$$

where

$$m_y=\int_{-h/2}^{h/2}(\bar{X}_2+\Delta X_2)x_3\mathrm{d}x_3+\frac{h}{2}\Big[(\sigma_{31}^++\sigma_{31}^-)u_{2,1}$$
$$+\frac{h}{2}(\sigma_{31}^+-\sigma_{31}^-)\psi_{2,1}+(\sigma_{32}^++\sigma_{32}^-)u_{2,2}+\frac{h}{2}(\sigma_{32}^+-\sigma_{32}^-)\psi_{2,2}$$
$$+(\sigma_{33}^++\sigma_{33}^-)\psi_2+\bar{\sigma}_{23}^++\bar{\sigma}_{23}^-\Big]$$

Boundary conditions are derived as the same procedures of governing equations, and the five boundary traction conditions are

$$\bar{F}_{nn}+\Delta F_{nn}=N_n u_{n,n}+M_n\psi_{n,n}+N_{nt}u_{n,t}+M_{nt}\psi_{n,t}+Q_n\psi_n$$
$$+D(u_{n,n}+\tfrac{1}{2}w_{,n}^2+\nu u_{t,t}+\tfrac{1}{2}\nu w_{,t}^2)$$
$$\bar{F}_{nt}+\Delta F_{nt}=N_n u_{t,n}+M_n\psi_{t,n}+N_{nt}u_{t,t}+M_{nt}\psi_{t,t}+Q_n\psi_t$$
$$+Gh(u_{n,t}+u_{t,n}+w_{,n}w_{,t})$$
$$\bar{F}_{n3}+\Delta F_{n3}=D(u_{n,n}+\tfrac{1}{2}w_{,n}^2+\nu u_{t,t}+\tfrac{1}{2}\nu w_{,t}^2)w_{,n}$$
$$+Gh(u_{n,t}+u_{t,n}+w_{,n}w_{,t})w_{,t}+N_n w_{,n}+N_{nt}w_{,t}$$
$$+\kappa^2G^*h(\psi_n+w_{,n}) \tag{11}$$

$$\bar{M}_{nn}+\Delta M_{nn}=M_n u_{n,n}+M_n^*\psi_{n,n}+M_{nt}u_{n,t}+M_{nt}^*\psi_{n,t}+Q_n^*\psi_n$$
$$+D^*(\psi_{n,n}+\nu\psi_{t,t})$$
$$\bar{M}_{nt}+\Delta M_{nt}=M_n u_{t,n}+M_n^*\psi_{t,n}+M_{nt}u_{t,t}+M_{nt}^*\psi_{t,t}+Q_n^*\psi_t$$
$$+\frac{D^*}{2}(1-\nu)(\psi_{n,t}+\psi_{t,n})$$

Alternative displacement boundary conditions are

$$u_n=u_{nn},\ u_t=u_{nt},\ w=w_{n3},\ \psi_n=\psi_{nn},\ \psi_t=\psi_{nt} \tag{12}$$

where the quantities on the right-hand sides are prescribed and the subscripts n and t denote the normal and tangential directions of the plate's edge, respectively.

Example Problem

We consider a simply supported rectangular plate in a state of arbitrary edge loading. The state of applied stress is

$$\sigma_{11}=\sigma_n+2x_3\sigma_m/h \tag{13}$$

σ_m and σ_n are taken to be constants. It is comprised of a compressive plus a bending stress (see Fig. 1). The only nonzero stress resultants are

$$N_x=h\sigma_n,\ M_x=h^2\sigma_m/6,\ M_x^*=h^3\sigma_n/12 \tag{14}$$

Lateral loads and body forces are taken to be zero

$$f_x,f_y,q,m_x,m_y=0 \tag{15}$$

The equations of motion [Eqs. (6-10)] are simplified to

$$M_x\psi_{1,11}+N_xu_{1,11}+D(u_{1,11}+\nu u_{2,11}+w_{,1}w_{,11}+\nu w_{,2}w_{,12})$$
$$+Gh(u_{1,22}+u_{2,12}+w_{,2}w_{,12}+w_{,1}w_{,22})=0$$
$$M_x\psi_{2,11}+N_xu_{2,11}+Gh(u_{1,21}+u_{2,11}+w_{,1}w_{,12}+w_{,2}w_{,11})$$
$$+D(u_{2,22}+\nu u_{1,12}+w_{,2}w_{,12}+\nu w_{,1}w_{,12})=0 \tag{16}$$
$$D(\epsilon_1+\nu\epsilon_2)w_{,11}+2Gh\gamma w_{,12}+D(\nu\epsilon_1+\epsilon_2)$$
$$+D(\epsilon_1+\nu\epsilon_2)_{,1}w_{,1}+Gh\gamma_{,2}w_{,1}+D(\nu\epsilon_1+\epsilon_2)_{,2}w_{,2}$$
$$+Gh\gamma_{,1}w_{,2}+N_xw_{,11}+\kappa^2G^*h(\psi_{1,1}+w_{,11})$$
$$+\kappa^2G^*h(\psi_{2,2}+w_{,22})=0$$
$$M_x^*\psi_{1,11}+M_xu_{1,11}+D^*(\psi_{1,11}+\nu\psi_{2,21})$$
$$+\frac{Gh^3}{12}(\psi_{1,22}+\psi_{2,12})-\kappa^2G^*h(\psi_1+w_{,1})=0$$

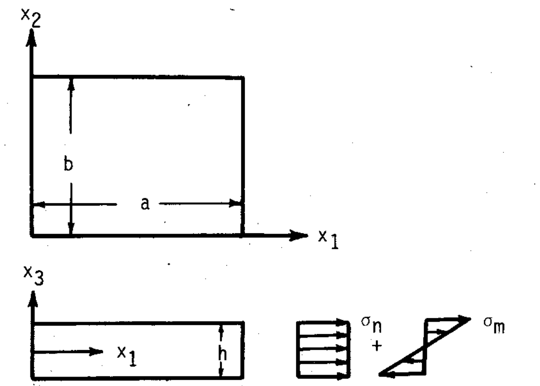

Fig. 1 The rectangular plate and the applied stress field.

$$M_x^*\psi_{2,11}+M_xu_{2,11}+\frac{Gh^3}{12}(\psi_{1,21}+\psi_{2,11})$$
$$+D^*(\psi_{2,22}+\nu\psi_{1,12})-\kappa^2G^*h(\psi_2+w_{,2})=0$$

For the simply supported plate the boundary conditions are, on the x_1 constant edges,

$$w=0,\quad \psi_2=0,\quad u_2=0$$
$$\bar{F}_{11}+\Delta F_{11}=M_x\psi_{1,1}+N_xu_{1,1}+D(u_{1,1}+\nu u_{2,2}$$
$$+\tfrac{1}{2}w_{,1}^2+\tfrac{1}{2}\nu w_{,2}^2)=0 \tag{17}$$
$$\bar{M}_{11}+\Delta M_{11}=M_x^*\psi_{1,1}+M_xu_{1,1}+D^*(\psi_{1,1}+\nu\psi_{2,2})=0$$

and, on the x_2 constant edges,

$$w=0,\quad \psi_1=0,\quad u_1=0$$
$$\bar{F}_{22}+\Delta F_{22}=M_y\psi_{2,2}+N_yu_{2,2}+D(u_{2,2}+\nu u_{1,1}$$
$$+\tfrac{1}{2}w_{,2}^2+\tfrac{1}{2}\nu w_{,1}^2)=0 \tag{18}$$
$$\bar{M}_{22}+\Delta M_{22}=M_y^*\psi_{2,2}+M_yu_{2,2}+D^*(\psi_{2,2}+\nu\psi_{1,1})=0$$

The following assumed displacement function will satisfy the geometric boundary conditions [Eqs. (17) and (18)]

$$u_1=\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}u_{1mn}\cos\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}$$
$$u_2=\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}u_{2mn}\sin\frac{m\pi x_1}{a}\cos\frac{n\pi x_2}{b}$$
$$w=\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}w_{mn}\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}$$
$$\psi_1=\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}\psi_{1mn}\cos\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}$$
$$\psi_2=\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}\psi_{2mn}\sin\frac{m\pi x_1}{a}\cos\frac{n\pi x_2}{b} \tag{19}$$

The three terms of Galerkin's approximate method are used to solve the nonlinear problems. The resulting nonlinear system equations are solved by Newton's algorithm method. All of the nondimensionalized parameters are listed in the nomenclature.

The buckling load K_{cr} can be obtained and the results are compared with Brunelle's solutions. The postbuckled transverse deflections are calculated and the effects of various parameters studied.

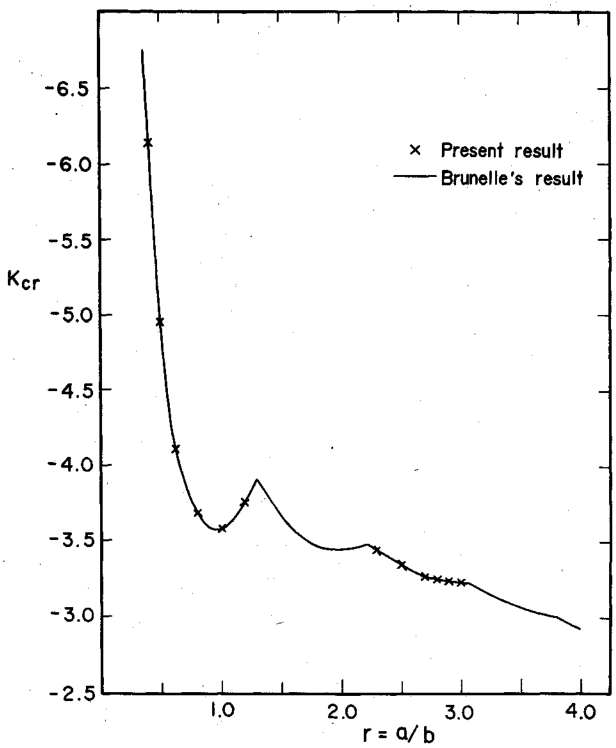

Fig. 2 Comparison of Brunelle's results with present results ($a/h = 10, S = 0.05, \beta = 0$).

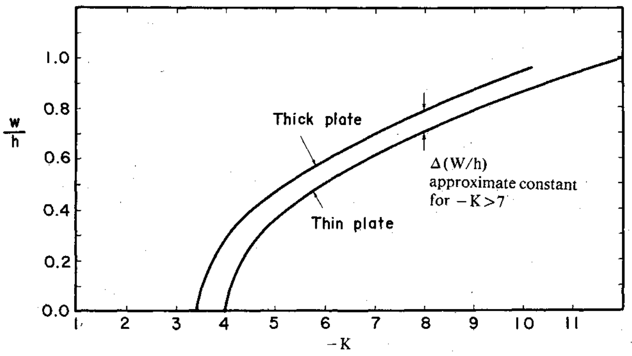

Fig. 3 Relation between postbuckling loads and central deflections for thick and thin plates (thick plate: $a/h = 10$, $S = 0.05$, $r = 1$, $\beta = 0$; thin plate: $a/h = 100$, $S = 0.0001$, $r = 1$, $\beta = 0$).

Results and Discussion

Many parameters have effects on the postbuckling behavior. For verifying the accuracy of present results, the buckling loads with no bending stresses are the first to be considered and the results ae compared with Brunelle's[12] results. Figure 2 plots K_{cr} for various aspect ratios of Brunelle's results: a/h, S, and β are 10, 0.05, and 0, respectively. It can be seen that the K_{cr} values calculated by the present formulations coincide with Brunelle's.

In the Fig. 3 plots of K vs W, a/h, S, and r are 10, 0.05, and 1 for a thick plate and 100, 0.0001, and 1 for a thin plate, respectively. The effects of shear deformation can be observed from the differences in the deflections between thick and thin plates. From Fig. 3, the deflections of the thick plate are larger than those of the thin plate and the differences are approximately constant for K larger than -7.0.

Figure 4 shows the effect of aspect ratio r. The deflection increases with the increasing aspect ratio. The relationship of S to W is shown in Fig. 5, where r, a/h, and β are equal to 1, 10, and 0, respectively. It is seen that the larger the transverse isotropic coefficient S is, the larger the deflection is. This

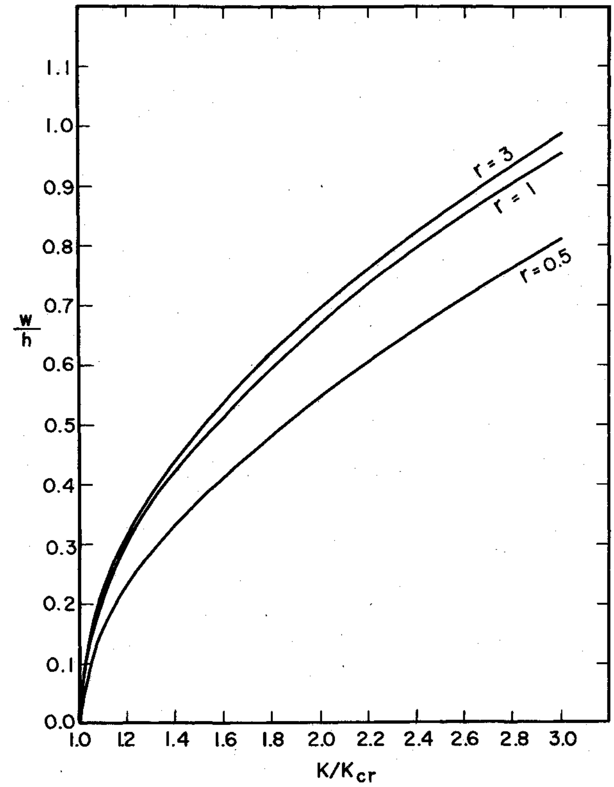

Fig. 4 Central deflection vs postbuckling load for various values of r ($a/h = 10, S = 0.05, \beta = 0$).

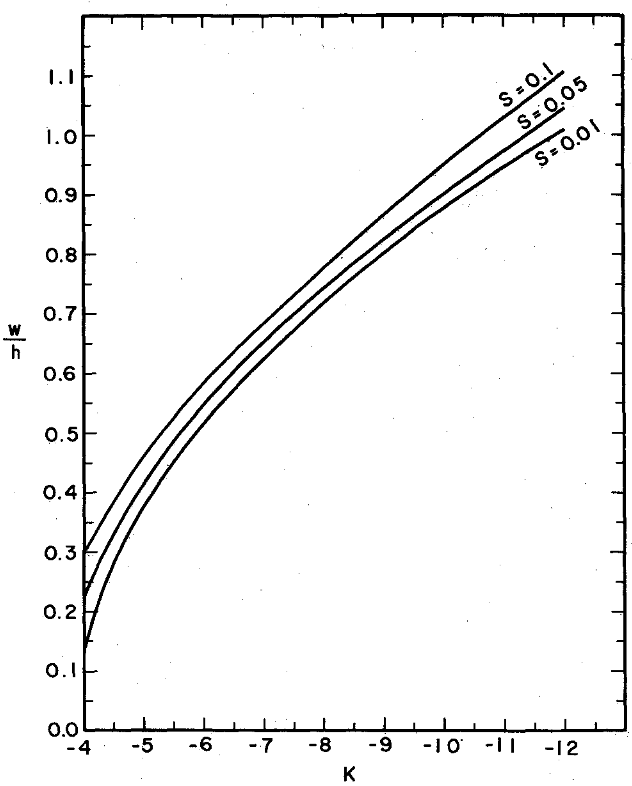

Fig. 5 Central deflection vs postbuckling load for various values of S ($a/h = 10, r = 1, \beta = 0$).

means that the postbuckled strength will decrease when the transverse shear resistant is small.

Plots of W vs K/K_{cr} for various values of β are shown in Fig. 6 where r, S, and a/h are equal to 1, 0.05, and 10, respectively. The bending stress effects are shown to increase the deflection when β is positive.

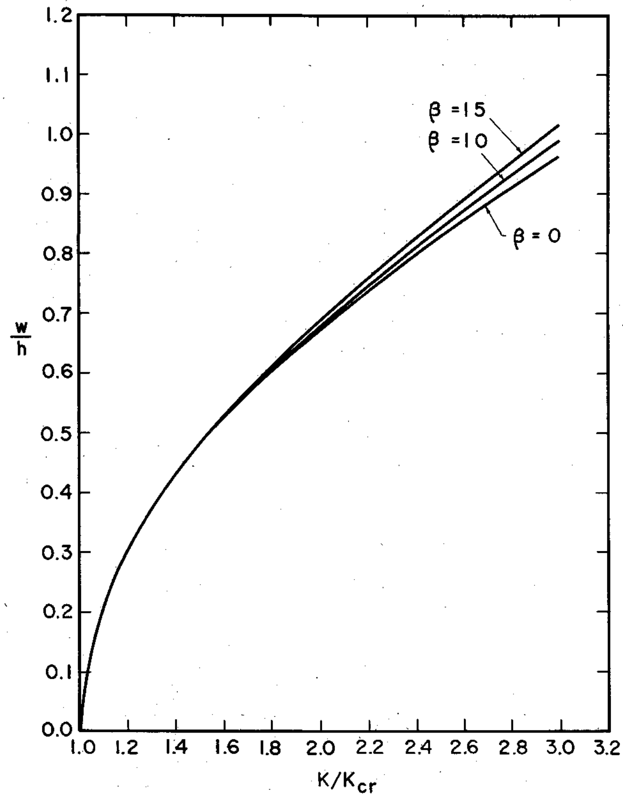

Fig. 6 Central deflection vs postbuckling load for various values of β ($a/h=10$, $S=0.05$, $r=1$).

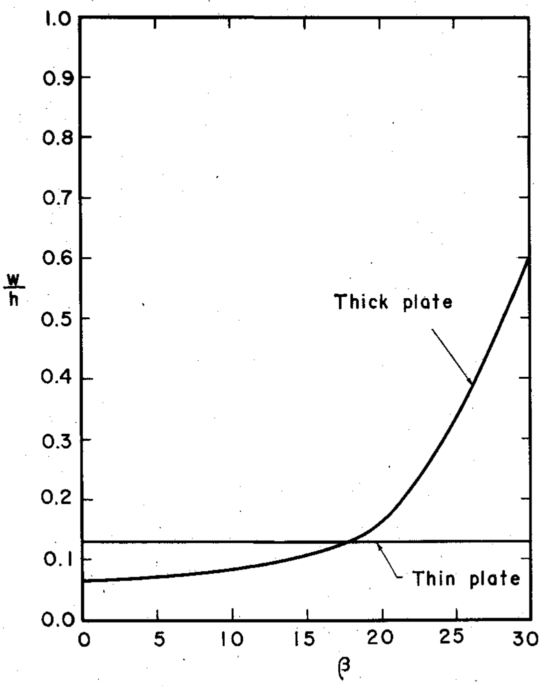

Fig. 7 Central deflection vs β for thick and thin plates when $K/K_{cr}=2.2$ (thick plate: $a/h=10$, $S=0.05$, $r=1$; thin plate: $a/h=100$, $S=0.0001$, $r=1$).

For comparison, the effect of bending stress on postbuckling behavior of thick and thin plates K/K_{cr} is chosen to 2.2. Plots of W vs β are made in Fig. 7 for thick plates and thin plates, respectively. It can be seen that the bending stress coefficient β has little effect on thin plates, but significant effects on thick plates.

Conclusions

The preliminary results indicate that:

1) Using the present method, it is possible to obtain the same buckling load as Brunelle.

2) The thick plate buckles at a lower $-K$ than the thin plate, and the postbuckled deflection of the thick plate is larger than that of the corresponding thin plate.

3) The postbuckled deflection of the plate increases when the aspect ratio increases.

4) Deflection increases with transverse isotropic coefficient.

5) The deflection decreases with the increasing bending stress coefficient; the effects are significant for thick plates.

The results presented do not cover all of the possible cases for this problem. However, they do indicate some of the many interesting effects that can be studied with the present equations. The postbuckling problems involving various boundary conditions and various edge-loading distributions of thick plates are still to be investigated. Particularly interesting are problems of the effects of nonconservative forces on the postbuckling behavior.

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References

[1]Coan, J. M., "Large Deflection Theory for Plates with Small Initial Curvature Loaded in Edge Compression," *Journal of Applied Mechanics,* Vol. 18, June 1951, pp. 143-151.

[2]Yamaki, N., "Post-Buckling Behavior of Rectangular Plates with Small Initial Curvatures Loaded in Edge Compression," *Journal of Applied Mechanics,* Vol. 26,Sept. 1959, pp. 407-414.

[3]Walker, A. C., "The Post-Buckling Behavior of Simply-Supported Square Plates," *The Aeronautical Quarterly,* Vol. 20, Aug. 1969, pp. 203-222.

[4]Rhodes, J. and Harvey, J. M., "Plates in Uniaxial Compression with Various Support Conditions at the Unloaded Boundaries," *International Journal of Mechanical Science,* Vol. 13, Sept. 1971, pp. 787-802.

[5]Pandalai, K. A. V. and Sathyammorthy, M., "Postbuckling Behavior of Orthotropic Skew Plates," *AIAA Journal,* Vol. 11, May 1973, pp. 731-733.

[6]Prabhakara, M. K. and Chia, C. Y., "Postbuckling Behavior of Rectangular Orthotropic Plates," *Journal of Mechanical Engineering Science,* Vol. 15, Feb. 1973, pp. 25-33.

[7]Chia, C. Y. and Prabhakara, M. K., "Postbuckling Behavior of Unsymmetrical Layered Anisotropic Rectangular Plates," *Journal of Applied Mechanics,* Vol. 41, March 1974, pp. 155-162.

[8]Fok, W. C. and Rhodes, J., "Postbuckling Behavior of Plates with Discontinuous Change of Thickness," *International Journal of Mechanical Science,* Vol. 19, Oct. 1977, pp. 603-618.

[9]Walker, A. C., "Flat Rectangular Plates Subjected to a Linearly-Varying Edge Compressive Loading," *Thin-Walled Structures,* edited by A. H. Chilver, Chatto and Windus, London, 1967, pp. 208-247.

[10]Uemura, M. and Byon, O., "Secondary Buckling of a Flat Plate under Uniaxial Compression," *International Journal of Non-Linear Mechanics,* Vol. 12, June 1977, pp. 355-370.

[11]Uemura, M. and Byon, O., "Secondary Buckling of a Flat Plate under Uniaxial Compression," *International Journal of Non-Linear Mechanics,* Vol. 13, Jan. 1978, pp. 1-14.

[12]Brunelle, E. J. and Robertson, S. R., "Initially Stressed Mindlin Plates," *AIAA Journal,* Vol. 12, Aug. 1974, pp. 1036-1044.

[13]Chen, L. W. and Doong, J. L., "Large Amplitude Vibration of an Initially Stressed Thick Plate," *Proceedings of NCKU/AAS International Symposium on Engineering Sciences and Mechanics,* Taiwan, Republic of China, Vol. 2, 1981, pp. 776-789.

[14]Bolotin, V. V., *Non-Conservative Problems of the Theory of Elastic Stability,* MacMillan Co., New York, 1963.